Tips for a Successful First Course in Calculus

So you've signed up to take a class in Calculus; I hope you will enjoy it! Perhaps you are a high school student whose primary motivation is to demonstrate the ability to succeed in a rigorous analytic course; maybe you want to see if a future STEM major in college is right for you. Or maybe you are already in college and hoping to earn your required math credits, or are striving to excel in a "weeder" course to get into upper level STEM classes.

I have taught students in all these circumstances and have a few tips on how to make sure you have the best chance of success.

Review of Pre-Calculus Essentials

One reason that Calculus can be intimidating is that it seems to require mastery of years of preparatory work in algebra – I barely remember what I had for breakfast, let along something I learned as a sophomore! Some students who find the concepts of calculus manageable don't earn the grade they'd like because of too many pre-calc mistakes. However, I want to reassure you that many students find calculus to be a rewarding class where many concepts from prior classes start to click.

If you have a week or two before the start of the semester, it is a great idea to shake off some rust and review some algebra, and it's not too late to do this after the course has started. Here's a breakdown of some things to focus on:

Polynomials and Rational Functions

You'll be happy to know that some of the more difficult tasks like **factoring** using the **rational root theorem** and **synthetic division**, or finding the exact equation of an **oblique asymptote** will not be required.

You will be expected to have some intuition for what features the graphs of these functions have, and the ability to manipulate the expressions algebraically, but you should not expect to have to deal with situations as complicated as you were tested on last year.

Trigonometry

The most important thing here is that you are comfortable with **radian** measure and know the trig function values of the special angles in each quadrant. Degree measure may be used in a diagram, or some story problem, but calculus work will be in radians.

You should be comfortable with the graphs of trig functions to the extent of knowing the basic shape, how to find periods, maxima and minima.

The reciprocal and quotient relationships between trig functions (e.g. $\tan x = \frac{\sin x}{\cos x}$ or $\sec x = \frac{1}{\cos x}$) as well as the Pythagorean identities (e.g. $\sin^2 x + \cos^2 x = 1$) will be important, but you will **not** have to remember all the different angle sum and difference formula (most classes will take time to review on the rare occasion one may be needed), or be asked to solve intricate trig equations.

Logs and Exponentials

These functions tend to give pre-calc students more difficulty simply because they are less familiar, but budding mathematicians begin to realize that they are actually easier to work with than many other types of functions. You should certainly know the algebraic **laws of logs** and exponents, though teachers will understand that it will take a little longer to develop intuition about them.

Pre-calculus classes will usually talk about the number $e \approx 2.718$ that can be used as a base of an exponential function, and the **natural logarithm,** $\ln(x)$, without making it totally clear why this log is natural, or what is so great about *e*. But much like with radian measure, the reason will become clearer in calculus. Going in, it is important to be comfortable with e^x and $\ln(x)$. They really are easier to use in calculus, so much so that you will likely use a change of base formula if given anything else.

Function Notation

Being comfortable with the notation and concepts of functions will be critical in calculus. There won't necessarily be as much focus on *transformations* like comparing the graph $y = f(x)$ to that of $y = f(x - h) + k$, but being fluent in considering the idea of a function from multiple perspectives (as a graph, as an algebraic rule, being comfortable with compositions, inverses, and substituting different input variable expressions) is important.

Here is one of the places where understanding the ideas of calculus can actually help you understand the point and importance of some of the abstract notations of pre-calc. So be patient with yourself and try to keep in mind the goal you are aiming for as you struggle with how to push the symbols around the page.

Other Topics

I'll give some good news here and list some of the things that you studied in pre-calculus that you will not necessarily need to use in a first Calculus course (perhaps unless you are taking an accelerated AP Calc BC)

- **Vectors** – You should not see them, nor their close relatives **Parametric Functions**. These topics will be important again in Calc 3, BC or other advanced courses.
- **Complex Numbers** – In pre-calc, these came up basically only as "imaginary" solutions to polynomial equations. In later math classes it will be important to be comfortable in handling the arithmetic of complex numbers, and to think about functions that use them as input or output, but they are basically ignored in an introductory Calculus class.
- **Sequences and Series** – You may have struggled with such things as arithmetic and geometric sequences, or even proofs by induction, in pre-calc. But these are usually not

used heavily in a first Calc course. However, being at least comfortable with **summation notation** is expected. Series will come back to play a large role in Calc BC.

Keep a broad view

It is easy to get lost in the minutia of a calculus class, but try not to let new notations and intricate rules of symbol manipulation complicate what comes down to just a few important foundational concepts. The biggest ideas of Calculus are:

- **The Derivative**: The rate of change of the output of a function with respect to the input. There are a huge number of applications of this idea to analyze functions and their graphs, and for real-life applications.
- **The Integral**: This is an "accumulation" of output of a function. Again, you will see many applications from geometry (area under a curve) to the sciences.
- **The Limit**: Many classes start with discussing limits, though it is a bit more abstract and tricky of a concept. In your first Calc class the most important use of the limit will be figuring out how to calculate integrals and derivatives.
- **The Fundamental Theorem of Calculus**: This explains a connection between the first two big ideas. It is instructive to think of a physics example. Consider a function giving position as a function of time. The derivative, i.e. rate of change of position with respect to time, is the velocity function. On the other hand, the integral, or "accumulation" of the velocity function will give the net displacement, the change in position. Perhaps this gives some idea of how the two main operations of calculus are connected; in some sense they are inverses of each other.

It is not possible to develop intuition for the whole subject in a short blog post, but I encourage you to find one of many quality video lectures on the subject. In my opinion, one of the best comes from the youtube channel 3Blue1Brown, but there are quite a few options out there. Go back to it later in the course if you start to lose a sense of the purpose of the course.

How to take notes

There are a couple of good reasons to take notes in a math class. During class time it can help you avoid having your attention wander (in some particularly boring classes – not math of course! – I even attempted taking notes left-handed just so that the extra challenge forced me to focus). Even if you never have to struggle against daydreaming, it is important to realize that in a math class you should not be passively receiving information, but rather actively attempting to think ahead. Try to predict the conclusion of an argument, or the steps of solving a problem. It is a common mistake to think that being able to follow a presented solution means that you will be able to do something similar by yourself later – you must go through a lot of practice problems, which your textbook or teacher/professor will provide. It is also hugely important to try the worked practice problems from your textbook, attempting them first on your own and then comparing your solution to how the problem was solved in the textbook.

Notes can also be useful for review, but only if they are well organized and neatly written. Some teachers will be extremely helpful and give daily note sets or worksheets, ideally in a skeleton form with blanks to fill and practice problems to solve. These are very useful for the class at hand, but a student will still need to develop the skill and habit of being able to take notes on their own, even with such aids.

When I am taking notes for math, I like to start each day with a blank white sheet of paper at the top of which I simply write the date, class name, and lecture number if I've been keeping track. I like to have loose sheets in a folder because it seems more efficient to me, but I understand that bound notebooks are also popular. I like unlined paper so that I can freely sketch graphs or diagrams, or larger math expressions. It should be clear where new ideas start, with an underlined label like **Def** (for a new definition) **Thm** (for a theorem), **Ex** (for an example problem), etc. Parts of the lecture where the instructor perhaps gives some of the motivation for an idea, or the history behind it, are usually parts where you can rest your pencil – unless you think it will be key to jog your memory when re-reading, you don't need to write down every word. But however you choose to take notes, it is essential you write down and understand everything from the lecture, that you restudy those lecture notes regularly, and that they are at the tips of your fingertips and completely accessible to you on exam day.

Good luck and have fun with calculus!